ON INVESTIGATION OF ISOMORPHISM OF DIFFERENT SOLUTIONS OF THE B.I.B. DESIGN (31, 31, 15, 15, 7)

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1. INTRODUCTION

For the B.I.B. design (31, 31, 15, 15, 7) the following solutions:

(*A*) : (0, 1, 2, 3, 5, 6, 8, 11, 12, 18, 19, 20, 23, 27, 29) through the cyclic solution to *PG* (4, 2) : 3,

(*B*) : (1, 2, 4, 5, 7, 8, 9, 10, 14, 16, 18, 19, 20, 25, 28) Bose (1939), and

(*C*) : (1, 2, 3, 4, 6, 8, 12, 15, 16, 17, 23, 24, 27, 29, 30) Marshal Hall (1958),

were considered by Rao (1961) and he established that the last two solutions are non-isomorphic. The relationship of these solutions to the first solution was, however, not investigated. In the present paper the relationship of these solutions has been investigated by the method of standardisation as given by Das and Kulshreshtha (1968). It has been found that the solutions *A* and *C* are isomorphic whereas *A* and *B* are non-isomorphic.

2. INVESTIGATION OF ISOMORPHISM

The method of standardisation is applicable for B.I.B. designs obtainable through one initial block. When v is a prime so that the elements of the field are residues modulo (v), the initial block can be presented with its first m elements, (where m is the maximum number of consecutive numbers in the initial block), as 1, 2, 3,...m by adding a suitable element of the field to the intial block elements. Such an initial block is said to be in the standard form. The method consists of getting (v-1) blocks obtained by multiplying an initial block by the nonzero elements of the field and presenting each one of them in the standard form. The different (isomorphic) initial blocks from the above $(v-1)$ blocks are Distinct Isomorphic Initial Blocks (DIIB).

Since the solutions A and C give six identical DIIB hence they are isomorphic. These DIIB are

1, 2, 3, 4, 6, 7, 9, 12, 13, 19, 20, 21, 24, 28, 30
1, 2, 3, 4, 7, 8, 10, 11, 17, 21, 22, 23, 25, 27, 30
1, 2, 3, 4, 8, 9, 12, 18, 20, 21, 22, 24, 25, 27, 29
1, 2, 3, 4, 6, 8, 12, 15, 16, 17, 23, 24, 27, 29, 30
1, 2, 3, 4, 6, 9, 11, 13, 14, 15, 19, 25, 26, 28, 29
1, 2, 3, 4, 7, 9, 11, 12, 14, 15, 16, 18, 24, 27, 28

The relationship of the Solutions A and C is the permutation $P=4-2^nQ$, mod (31), where P denotes varietal numbers in the solution A, Q denotes varietal numbers in the solution C and $n=0, 1, 2, 3$ or 4. Since the solutions B and C are nonisomorphic, [Rao (1961)] A and B also must be nonisomorphic.

Further, there are only two DIIB for the solution B, *viz* :

1, 2. 3, 4, 8, 10, 12, 13, 14, 19, 22, 26, 27, 29, 30
1, 2, 3, 4, 6, 7, 9, 10, 14, 17, 22, 23, 24, 26, 28

It is easy to prove that there are only two DIIB for the series of B.I.B. designs with $v=4\lambda+3$ and $k=2\lambda+1$ obtainable by Bose's method. These two DIIB correspond to the even and odd powers of the primitive root of the field.

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